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SCATTERING OF A PLANE ELECTROMAGNETIC WAVE FROM A SEMICIRCULAR CRACK IN A PERFECTLY CONDUCTING GROUND PLANE

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The scattering of TM and TE plane electromagnetic waves from a semicylindrical crack in a perfectly conducting ground plane is determined using a dual-series eigenfunction expansion technique. Orthogonality properties of the sinusoids are used to reduce the solution to the inversion of a truncated infinite matrix. When the radius of the crack is much smaller than the wavelength, small-argument approximations for the spherical radial functions are employed to simplify the results and remove the ka-dependence from the necessary numerical matrix inversion. In this small crack limit, closed-form results are developed for both polarizations, and for the TM polarization, results are found to be different by only a constant factor from those for TM scattering from a narrow slit in an infinitely thin conducting screen. TE scattering from a small crack is found to differ significantly from TE scattering from a narrow slit, which indicates that the slit approximation is not valid for real situations that more closely resemble physically an indentation, or crack.

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Preface

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1. INTRODUCTION

The problem of scattering from, and coupling through, cracks and gaps arises when considering antennas and scatterers assembled from many subsections that meet to form cracks and gaps.

The *crack* problem has been studied by many people including Lord Rayleigh, who considered the problem of plane wave scattering from a half-cylindrical indentation in a ground plane.[1] In this paper he actually solved the half-cylindrical excrescence (protuberance) problem, with some discussion of experimental techniques related to the indentation, or crack, problem. The related problem of scattering from a slit in an infinitely thin screen has

[1] Lord Rayleigh (1907) On the light dispersed from fine lines ruled upon reflecting surfaces or transmitted by very narrow slits, *Phil Mag*, 14:350.

been extensively studied,[2, 3, 4] as has the scattering from a slit in a thick screen.[5, 6, 7, 8] Integral equations, experimental methods, and pure numerical techniques, such as finite-difference time domain, have been used to study the scattering from indentations in a ground plane.[9, 10, 11] The related problem of scattering from a cavity-backed slit has been considered recently using dual-series techniques.[12]

This report presents a dual-series eigenfunction approach to scattering of electromagnetic plane waves from a half-cylindrical (circular) indentation in a ground plane. We use to our advantage the coincidence of the problem geometry and a constant coordinate surface in circular cylindrical coordinates. The main difference between our analysis and typical eigenfunction problems lies in the incomplete orthogonality of the sinusoids over a half space, and the use of two separate regions of the problem.

Usefulness of the solution is primarily in the small-crack results, which are valid in the case where the radius of the crack is much smaller than the wavelength of the incident plane wave. For both the transverse magnetic and transverse electric polarizations, the ka-dependence can be removed from the necessary numerical matrix inversion when the crack is small. The matrix inversion need only be performed once for each polarization and it results in a constant coefficient for each case. Since the small-crack results are then functions of ka, the two results (TM and TE) can be used to consider cases where the incident field is not normal to the axis of the small crack.

[2] Bouwkamp, C.J. (1954) Diffraction theory, *Rep Prog Phys*, **17**:35.

[3] Chow, T.Y., and Adams, A.T. (1977) The coupling of electromagnetic waves through long slots, *IEEE T-EMC*, **19**:65.

[4] Butler, C.M., Rahmat-Samii, Y., and Mittra, R. (1978) Electromagnetic penetrations through apertures in conducting surfaces, *IEEE T-EMC*, **20**:82.

[5] Kashyap, S.C., and Hamid, M.A.K. (1971) Diffraction characteristics of a slit in a thick conducting screen, *IEEE T-AP*, **19**(No. 4):499.

[6] Hongo, K., and Ishii, G. (1978) Diffraction of an electromagnetic plane wave by a thick slit, *IEEE T-AP*, **26**(No. 3):494.

[7] Harrington, R.F., and Auckland, D.T. (1980) Electromagnetic transmission through narrow slots in thick conducting screens, *IEEE T-AP*, **28**:120.

[8] Taflove, A., Umashankar, K.R., Beker, B., Harfoush, F., and Yee, K.S. (1988) Detailed FD-TD analysis of electromagnetic fields penetrating narrow slots and lapped joints in thick conducting screens, *IEEE T-AP*, **36**(No. 2):247.

[9] White, G.S., and Feldman, A. (1981) Diffraction from a shallow rectangular groove, *Applied Optics*, **20**(No. 14):2585.

[10] White, G.S., and Marchiando, J.F. (1983) Scattering from a V-shaped groove in the resonance domain, *Applied Optics*, **22**(No. 15):2303.

[11] Dominek, A.K., Shamanski, H.T., and Wang, N. (1988) *Scattering from Three-Dimensional Cracks*, Ohio State University (preprint).

[12] Ziolkowski, R.W., and Grant, J.B. (1987) Scattering from cavity-backed aperture, *IEEE T-AP*, **35**(No. 5):504.

2. TRANSVERSE MAGNETIC POLARIZATION

2.1   Problem Geometry

We consider a Transverse Magnetic (TM) plane wave that is polarized in the z-direction and makes an angle ϕ^{inc} with the positive x-semiaxis. It is incident upon and scatters from a half-cylindrical indentation or crack in the ground plane, as shown in Figure 1.
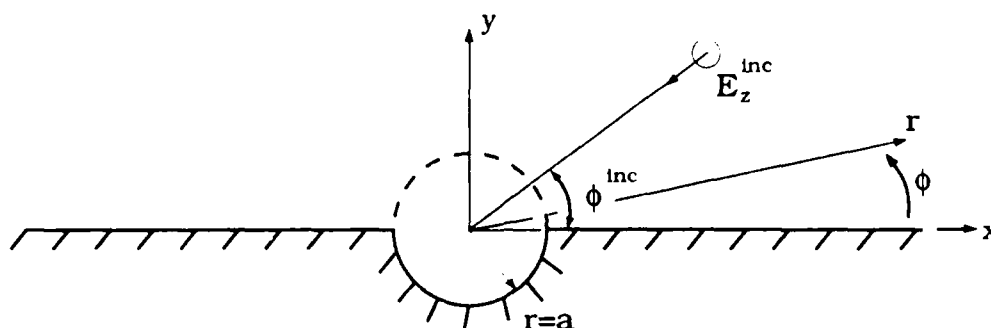

Figure 1. Problem Geometry for Incident TM Waves

The crack is of radius $r = a$ and of infinite extent in the z-direction. All angles are measured positive in the counterclockwise direction starting with $\phi = 0$ along the positive x-semiaxis. Angles are in the range $0 < \phi < 2\pi$. The crack is thus described by $\pi < \phi < 2\pi$ and $r = a$. The surface $0 < \phi < \pi$ and $r = a$ is referred to as the aperture. It is the region complementary to the crack. The problem will be formulated so that boundary conditions along the ground plane will be automatically satisfied, and then three boundary conditions will be applied at the surface $r = a$: one on the crack and two across the aperture. We define the $r > a$ to be the exterior region and $r < a$ to be the interior region, and the fields in these regions are the exterior and interior fields respectively.

2.2   Exterior Field

The incident field is a plane wave as discussed above. We write

$$E_z^{inc} = e^{ikr\cos\left(\phi - \phi^{inc}\right)},$$

(1)

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where unit amplitude has been assumed and $e^{i\omega t}$ time dependence suppressed. This plane wave has the following well-known expansion in terms of cylindrical Bessel functions [13]

$$E_z^{inc} = \sum_{n=-\infty}^{\infty} i^n J_n(kr) e^{in\left(\phi - \phi^{inc}\right)}.$$ (2)

In the exterior region the scattered field will be considered to be made up of two parts. The first is the reflected wave that would be present if there were no crack, and the second is the deviation from this caused by diffraction at the crack. Thus, in the exterior region we refer to the incident, reflected, and diffracted waves. We are, of course, most interested in the diffracted field.

The reflected wave is also a plane wave and is well-known. We have $\phi^{ref} = 2\pi - \phi^{inc}$ so that the reflected plane wave can be written as

$$E_z^{ref} = - e^{ikr \cos\left(\phi + \phi^{inc}\right)}.$$ (3)

In terms of cylindrical Bessel functions we have

$$E_z^{ref} = - \sum_{n=-\infty}^{\infty} i^n J_n(kr) e^{in\left(\phi + \phi^{inc}\right)}.$$ (4)

For the diffracted wave in the exterior region ($r \geq a$, $0 \leq \phi \leq \pi$) we write

$$E_z^{dif} = \sum_{n=0}^{\infty} A_n H_n^{(2)}(kr) \sin n\phi .$$ (5)

where A_n is an unknown modal coefficient and we have expanded this outgoing wave in terms of Hankel functions (of the second kind) and also $\sin n\phi$. Noting that because $\sin 0 = \sin n\pi = 0$, the diffracted wave vanishes on the ground plane and so satisfies the boundary condition there. Calling the interior Region 1 and the exterior Region 2, the electric fields become respectively, E^1 and E^2. If we write the total field in the exterior region as

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[13] Morse, P.M., and Feshbach, H. (1953) *Methods of Theoretical Physics*, McGraw-Hill Book Company, New York.

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$$E_z^2 = E_z^{inc} + E_z^{ref} + E_z^{dif},$$

(6)

we see that $E_z^2 = 0$ for $\phi = 0, \pi$ that is, on the ground plane.

2.3 Interior Field

In the interior region the total E_z-field can be written

$$E_z^1 = \sum_{n=-\infty}^{\infty} D_n J_n(kr) e^{in\phi}.$$

(7)

where D_n is an unknown modal coefficient. However, it will be more convenient for our analysis if we write this as

$$E_z^1 = \sum_{n=0}^{\infty} J_n(kr) (B_n \cos n\phi + C_n \sin n\phi).$$

(8)

where B_n and C_n are the modal coefficients that will be determined from the boundary conditions.

2.4 Total Exterior Field

We now wish to write the incident and reflected E_z-fields as series over $n = 0$ to ∞ rather than over all n. We note that $J_{-n}(kr) = (-1)^n J_n(kr)$.

$$E_z^{inc} = -J_0(kr) + \sum_{n=0}^{\infty} i^n J_n(kr) e^{+in(\phi - \phi^{inc})} + \sum_{n=0}^{\infty} i^n J_n(kr) e^{-in(\phi - \phi^{inc})}.$$

(9)

or

$$E_z^{inc} = -J_0(kr) + 2 \sum_{n=0}^{\infty} i^n J_n(kr) \cos n(\phi - \phi^{inc}).$$

(10)

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Similarly for the reflected wave we write

$$E_z^{ref} = J_0(kr) - 2 \sum_{n=0}^{\infty} i^n J_n(kr)\cos n\left(\phi + \phi^{inc}\right).$$

(11)

Hence,

$$E_z^{inc} + E_z^{ref} = 2 \sum_{n=0}^{\infty} i^n J_n(kr)\left[\cos n\left(\phi - \phi^{inc}\right) - \cos n\left(\phi + \phi^{inc}\right)\right].$$

(12)

If we use the trigonometric identity $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$ we can write

$$E_z^{inc} + E_z^{ref} = 4 \sum_{n=0}^{\infty} i^n J_n(kr)\sin n\phi^{inc}\sin n\phi.$$

(13)

Recalling the diffracted field, we have

$$E_z^2 = \sum_{n=0}^{\infty} \left[4 i^n J_n(kr)\sin n\phi^{inc} + A_n H_n^{(2)}(kr)\right]\sin n\phi,$$

(14)

for the exterior field. In this expression, A_n is the modal coefficient that we desire.

2.5 Magnetic Field Component

The ϕ-component of the magnetic field is also of interest in this problem. It is related to E_z by

$$H_\phi = \frac{-i}{\omega\mu} \partial_r E_z,$$

(15)

so that the exterior and interior H_ϕ-fields are

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$$H_\phi^2 = \frac{-ik}{\omega\mu} \sum_{n=0}^{\infty} \left[4 i^n J_n'(kr) \sin n\phi^{inc} + A_n H_n^{(2)'}(kr) \right] \sin n\phi.$$ (16)

and

$$H_\phi^1 = \frac{-ik}{\omega\mu} \sum_{n=0}^{\infty} J_n'(kr) \left(B_n \cos n\phi + C_n \sin n\phi \right).$$ (17)

where $J_n'(z) = \partial_z J_n(z)$.

2.6 Boundary Conditions

The required boundary conditions for solving this TM problem are

$$E_z^2 = 0 \text{ for } r = a \text{ and } \pi < \phi < 2\pi,$$ (18)

$$E_z^2 = E_z^1 \text{ for } r = a \text{ and } 0 < \phi < \pi,$$ (19)

$$H_\phi^2 = H_\phi^1 \text{ for } r = a \text{ and } 0 < \phi < \pi.$$ (20)

The first condition enforces zero tangential electric field on the surface of the perfectly reflecting crack and the last two ensure continuous tangential electric and magnetic fields across the aperture, or (imaginary) surface that is complementary to the crack.

The boundary conditions give the following three equations

$$\sum_{n=0}^{\infty} J_n(ka)B_n \cos n\phi_1 + \sum_{n=0}^{\infty} J_n(ka)C_n \sin n\phi_1 = 0.$$ (21)

$$\sum_{n=0}^{\infty} J_n(ka)B_n \cos n\phi_2 + \sum_{n=0}^{\infty} J_n(ka)C_n \sin n\phi_2 \tag{22}$$

$$= \sum_{n=0}^{\infty} \left[4i^n J_n(ka) \sin n\phi_2^{inc} + A_n H_n^{(2)}(ka) \right] \sin n\phi_2.$$

$$\sum_{n=0}^{\infty} J_n'(ka)B_n \cos n\phi_2 + \sum_{n=0}^{\infty} J_n'(ka)C_n \sin n\phi_2 \tag{23}$$

$$= \sum_{n=0}^{\infty} \left[4i^n J_n'(ka) \sin n\phi_2^{inc} + A_n H_n^{(2)'}(ka) \right] \sin n\phi_2.$$

where we must keep in mind that Eq. (21) is valid for $\pi < \phi_1 < 2\pi$ and Eqs. (22) and (23) are valid for $0 < \phi_2 < \pi$.

Now define the following for convenience

$$F_n = 4i^n J_n(ka) \sin n\phi^{inc} + A_n H_n^{(2)}(ka) - C_n J_n(ka), \tag{24}$$

$$G_n = B_n J_n(ka), \tag{25}$$

$$R_n = -C_n J_n(ka), \tag{26}$$

as well as $F'_n = \dfrac{\partial F_n}{\partial(ka)}$ and so on. The Boundary Condition equations are then

$$\sum_{n=0}^{\infty} G_n \cos n\phi_1 = \sum_{n=0}^{\infty} R_n \sin n\phi_1. \tag{27}$$

for $\pi < \phi_1 < 2\pi$ and

$$\sum_{n=0}^{\infty} G_n \cos n\phi_2 = \sum_{n=0}^{\infty} F_n \sin n\phi_2. \tag{28}$$

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$$\sum_{n=0}^{\infty} G'_n \cos n\phi_2 = \sum_{n=0}^{\infty} F'_n \sin n\phi_2. \qquad (29)$$

for $0 < \phi_2 < \pi$.

In Eq. (27) we make the change of variables $\Phi = \phi - \pi$ so that we have

$$\sum_{n=0}^{\infty} G_n \cos n(\Phi + \pi) = \sum_{n=0}^{\infty} R_n \sin n(\Phi + \pi), \qquad (30)$$

for $0 < \Phi < \pi$. Noting that $\cos n(\theta + \pi) = (-1)^n \cos n\theta$ and $\sin n(\theta + \pi) = (-1)^n \sin n\theta$ we have

$$\sum_{n=0}^{\infty} (-1)^n G_n \cos n\Phi = \sum_{n=0}^{\infty} (-1)^n R_n \sin n\Phi, \qquad (31)$$

for $0 < \Phi < \pi$. Thus, we see that all three Boundary Condition equations are of the form

$$\sum_{n=0}^{\infty} G_n \cos n\phi = \sum_{n=0}^{\infty} F_n \sin n\phi, \qquad (32)$$

where $0 < \phi < \pi$.

2.7 Orthogonality

We now make use of the following orthogonality relations among the sinusoids

$$\int_0^{\pi} \sin n\phi \sin m\phi \, d\phi = \frac{\pi}{2} \delta_{mn} \qquad m, n \neq 0, \qquad (33)$$

$$\int_0^{\pi} \sin n\phi \cos m\phi \, d\phi = \begin{cases} 0 & \text{if } n\text{-}m \text{ is even} \\ \\ \dfrac{2n}{n^2 - m^2} & \text{if } n\text{-}m \text{ is odd} \end{cases} \qquad (34)$$

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We multiply Eq. (32) by $\sin m\phi$ and integrate from 0 to π,

$$\int_0^\pi \sum_{n=0}^\infty G_n \cos n\phi \sin m\phi \, d\phi = \int_0^\pi \sum_{n=0}^\infty F_n \sin n\phi \sin m\phi \, d\phi. \tag{35}$$

Exchanging the sums and integrals and applying the orthogonality relations gives

$$2\sum_n^* \left(\frac{mG_n}{m^2 - n^2} \right) = \frac{\pi}{2} F_m, \tag{36}$$

where $\sum_n^*$ is used to indicate a sum over n from zero to infinity where $(n - m)$ is odd. Writing the even-odd behavior explicitly.

$$F_{m_e} = \frac{4}{\pi} \sum_{n=1,3,5}^\infty \frac{mG_n}{m^2 - n^2}, \tag{37}$$

$$F_{m_o} = \frac{4}{\pi} \sum_{n=0,2,4}^\infty \frac{mG_n}{m^2 - n^2}. \tag{38}$$

Referring to our previous definitions for F_n, G_n and R_n we write

$$4i^m J_m(ka) \sin m\phi^{inc} + A_m H_m^{(2)}(ka) - C_m J_m(ka) = \frac{4m}{\pi} \sum_n^* \frac{B_n J_n(ka)}{m^2 - n^2}, \tag{39}$$

$$4i^m J'_m(ka) \sin m\phi^{inc} + A_m H_m^{(2)'}(ka) - C_m J'_m(ka) = \frac{4m}{\pi} \sum_n^* \frac{B_n J'_n(ka)}{m^2 - n^2}, \tag{40}$$

and

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$$C_m J_m(ka) = \frac{4m}{\pi} \sum_n \frac{B_n J_n(ka)}{m^2 - n^2} \, . \tag{41}$$

These three equations can now be used to solve for the three modal coefficients A_n, B_n and C_n.

2.8 Solution for Modal Coefficients

Although the modal coefficients of interest in this problem are A_n, it is not possible to solve for them directly. Rather, we use Eqs. (39) and (40) to eliminate A_n, and after some algebra use Eq. (41) to eliminate C_n. The resulting expression may then be solved for B_n, and given that, we merely backsubstitute to get A_n.

Solving Eqs. (39) and (40) for A_n and equating them gives the relation

$$-4i^m J_m(ka) H_m^{(2)'}(ka) \sin m\phi^{inc} + C_m J_m(ka) H_m^{(2)'}(ka) \tag{42}$$

$$+ H_m^{(2)'}(ka) \frac{4m}{\pi} \sum_n \frac{B_n J_n(ka)}{m^2 - n^2}$$

$$= -4i^m J_m'(ka) H_m^{(2)}(ka) \sin m\phi^{inc} + C_m J_m'(ka) H_m^{(2)}(ka)$$

$$+ H_m^{(2)}(ka) \frac{4m}{\pi} \sum_n \frac{B_n J_n'(ka)}{m^2 - n^2} \, .$$

If we rearrange and use the Wronskian relationship, $W\left[J_m(ka) H_m^{(2)}(ka)\right] = \frac{-2i}{\pi ka}$, we can write

$$\left(4i^m \sin m\phi^{inc} - C_m\right) \frac{2i}{\pi ka} \tag{43}$$

$$= \frac{4m}{\pi} \sum_n \frac{B_n}{m^2 - n^2} \left[J_n'(ka) H_m^{(2)}(ka) - J_n(ka) H_m^{(2)'}(ka) \right] \, .$$

Substituting for C_n we find

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$$4i^m \sin m\phi^{inc} = \frac{4m}{\pi} \sum_n \frac{B_n}{m^2 - n^2} \frac{J_n(ka)}{J_m(ka)} \tag{44}$$

$$+ \left(\frac{\pi ka}{2i}\right) \frac{4m}{\pi} \sum_n \frac{B_n}{m^2 - n^2} \left[J_n'(ka) H_m^{(2)}(ka) - J_n(ka) H_m^{(2)}(ka) \right].$$

Finally, rearranging this gives

$$\sum_n \left(\frac{B_n}{m^2 - n^2}\right) \left\{ J_n(ka) + i\pi \left(\frac{ka}{2}\right) \left[J_n(ka) H_m^{(2)'}(ka) J_m(ka) - J_n'(ka) H_m^{(2)}(ka) J_m(ka) \right] \right\} \tag{45}$$

$$= \frac{\pi}{m} i^m J_m(ka) \sin m\phi^{inc}$$

which we can solve for B_n . Once the B_n are found, the A_l can be found from

$$A_l = \left\{ \frac{8l}{\pi} \sum_n \left(\frac{B_n J_n(ka)}{l^2 - n^2}\right) - 4i^l J_l(ka) \sin l\phi^{inc} \right\} \frac{1}{H_l^{(2)}(ka)} . \tag{46}$$

which determines the diffracted field in the exterior region.

2.9   Small Crack Limit

When $ka \to 0$ the cylinder functions have the following simple forms

$$J_l(ka) \to \frac{(ka/2)^l}{l!} . \tag{47}$$

and

$$H_l^{(2)}(ka) \to \frac{i}{l\pi} \frac{l!}{(ka/2)^l} . \tag{48}$$

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The equations that we must solve also become much simpler, and we can separate the ka-dependence from the necessary numerical matrix inversion.

When the radius of the crack is small compared to the wavelength of the incident TM wave we have, for small $\varepsilon = \dfrac{ka}{2}$,

$$\sum_n^{\bullet} \left(\frac{B_n \varepsilon^n / n!}{m^2 - n^2} \right) \left[\frac{3m+n}{2} \right] = \pi i^m \frac{\varepsilon^m}{m!} \sin m\phi^{inc}, \tag{49}$$

and

$$A_l = 4 \, il \frac{\varepsilon^l}{l!} \left[\pi i^l \frac{\varepsilon^l}{l!} \sin l\phi^{inc} - \sum_n^{\bullet} \left(\frac{2 \, l B_n \varepsilon^n / n!}{l^2 - n^2} \right) \right]. \tag{50}$$

We define the following

$$f_m = \frac{\pi i^m}{m!} \sin m\phi^{inc}, \tag{51}$$

$$T_{mn} = \frac{3m+n}{2\left(m^2 - n^2\right)}, \tag{52}$$

$$b_n = \frac{B_n \varepsilon^n}{n!}. \tag{53}$$

We then have

$$\sum_n^{\bullet} T_{mn} b_n = f_m \varepsilon^m, \tag{54}$$

$$A_l = 4 \, il \frac{\varepsilon^l}{l!} \left[f_l \varepsilon^l - 2 \, l \sum_n^{\bullet} \frac{b_n}{l^2 - n^2} \right]. \tag{55}$$

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Hence, we write

$$A_l = 4 \, il \frac{\varepsilon^l}{l!} \left[f_l \varepsilon^l - 2 \, l \sum_n^{\bullet} \sum_{m=1}^{\infty} \frac{T_{nm}^{-1} f_m \varepsilon^m}{l^2 - n^2} \right] = 4 \, il \frac{\varepsilon^l}{l!} \sum_{m=1}^{\infty} \mathcal{G}_{lm} f_m \varepsilon^m, \tag{56}$$

where

$$\mathcal{G}_{lm} = \delta_{lm} - 2 \, l \sum_n^{\bullet} \frac{T_{nm}^{-1}}{l^2 - n^2}. $$

For small ε (that is, cracks much narrower than a wavelength) this shows that the only significant A_l is A_1, given approximately by

$$A_1 \approx -4 \, \pi \varepsilon^2 \mathcal{G}_{11} \sin \phi^{inc}. \tag{57}$$

We see from the above that the diffracted field behaves as $(ka)^2$ for small ka,

$$E_z^{dif} \approx -\mathcal{G}_{11} \pi (ka)^2 H_1^{(2)}(kr) \sin \phi \sin \phi^{inc}, \tag{58}$$

and

$$H_\phi^{dif} \approx i \mathcal{G}_{11} \sqrt{\frac{\varepsilon}{\mu}} \, \pi (ka)^2 H_1^{(2)'}(kr) \sin \phi \sin \phi^{inc}. \tag{59}$$

If we use the far-field expressions for the Hankel functions, we write for plane TM wave scattering from a small cylindrical crack ($r \to \infty$),

$$E_z^{dif} \approx -\mathcal{G}_{11} \sqrt{2\pi} \, (ka)^2 e^{i3\pi/4} \frac{e^{-ikr}}{\sqrt{kr}} \sin \phi \sin \phi^{inc}. \tag{60}$$

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and

$$H_\phi^{dif} \sim \sqrt{\frac{\varepsilon}{\mu}} \, E_z^{dif}. \tag{61}$$

The problem of plane wave scattering from a slit in an infinitely thin conducting plane has been solved, and its solution is well known. It has been discussed in a recent paper [14] and for this polarization and the special case of a narrow slit, we have

$$E_z^{dif} \Big|_{slit} = \frac{\sqrt{2\pi}}{4} \, (ka)^2 \, e^{i3\pi/4} \, \frac{e^{-ikr}}{\sqrt{kr}} \sin \phi \sin \phi^{inc}. \tag{62}$$

which is the same form as for the crack, but differing by a constant factor $\dfrac{-1}{4(\mathcal{Q}_{11})}$. Performing the necessary matrix inversion we find that $\mathcal{Q}_{11} \approx -0.19$ which gives for TM scattering from a small crack

$$E_z^{dif} = 0.19\pi \, (ka)^2 H_1^{(2)}(ka) \sin \phi \sin \phi^{inc}. \tag{63}$$

To check the consistency of the small-crack approximation, we have written a computer program to solve the general-ka Eqs., (45) and (46), and then compared the *general* results to the *small-crack* results as ka approaches zero. For small ka the first mode dominates, and a comparison for the two cases is shown in Figure 2. It can be seen that as the crack becomes small, the closed-form small-crack results coalesce with the general results.

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[14] Shore, R.A., and Yaghjian, A.D. (1988) Incremental diffraction coefficients for planar surfaces, *IEEE T-AP*, **36**(No. 1):55.
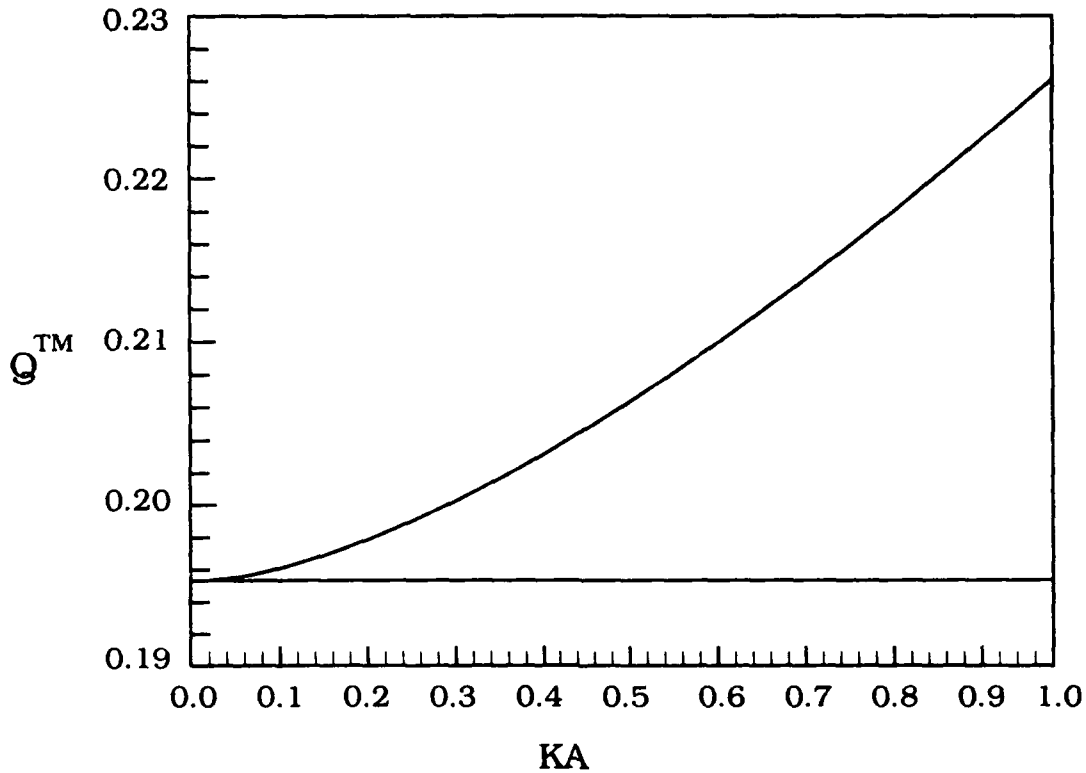
Figure 2. General and Small-Crack Q^{TM} vs. ka

3. TRANSVERSE ELECTRIC POLARIZATION

3.1   Problem Geometry

We consider a TE plane wave that is polarized in the z-direction and makes an angle ϕ^{inc} with the positive x-semiaxis. It is incident upon and scatters from a half-cylindrical indentation in the ground plane (see Figure 3). The crack is described by $r = a$ and $\pi < \phi < 2\pi$. It is of infinite extent in the z-direction and the problem is two dimensional. All angles are measured positive in the counterclockwise direction starting with $\phi = 0$ along the positive x-semiaxis. Angles are in the range $0 < \phi < 2\pi$.
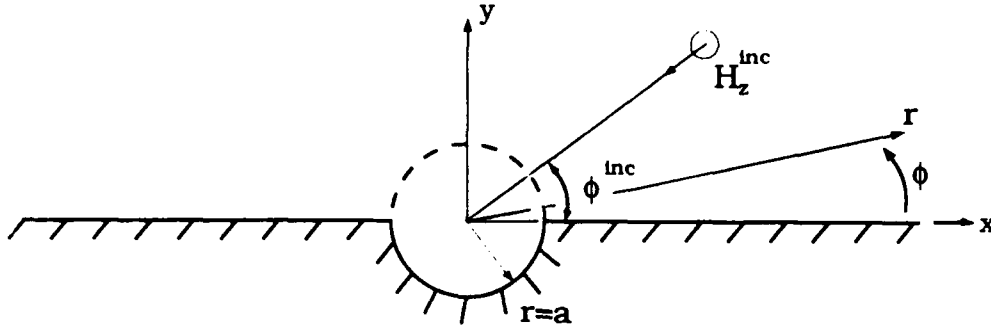
16

Figure 3. Problem Geometry for Incident TE Waves.

3.2 Exterior Field

The incident magnetic field is a plane wave written as

$$H_z^{inc} = e^{ikr\cos\left(\phi - \phi^{inc}\right)},$$

(64)

where unit amplitude has been assumed and $e^{i\omega t}$ time dependence suppressed. Written in cylindrical Bessel functions,[13] we have

$$H_z^{inc} = \sum_{n=-\infty}^{\infty} i^n J_n(kr) \qquad {}^{inc}\Big)$$

(65)

In the exterior region, the scattered field will be considered to be made up of two parts. The first is the reflected wave that would be present if there were no crack, and the second is the deviation from this caused by diffraction at the crack. Thus, in the exterior region we refer to the incident, reflected, and diffracted waves. We are most interested in the diffracted field.

The reflected wave is a plane wave and is well known. We have $\phi^{ref} = 2\pi - \phi^{inc}$ so that the reflected plane wave can be written as

$$H_z^{ref} = e^{ikr\cos\left(\phi + \phi^{inc}\right)}.$$

(66)

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In terms of cylindrical Bessel functions we have

$$H_z^{ref} = \sum_{n=-\infty}^{\infty} i^n J_n(kr)\, e^{in(\phi + \phi^{inc})}.$$

(67)

A bit of algebra and a few trigonometric identities allow us to show the sum of the incident and reflected H_z-waves satisfy the boundary condition of zero tangential electric field on the perfectly conducting ground plane.

For the diffracted H_z-wave we write

$$H_z^{dif} = \sum_{n=0}^{\infty} A_n H_n^{(2)}(kr)\, \cos n\phi.$$

(68)

where A_n is an unknown modal coefficient, and we have expanded this outgoing wave in terms of cylindrical Hankel functions (of the second kind) as well as $\cos n\phi$. Noting that because $\partial_\phi \cos n\phi = -n \sin n\phi$ and $\sin 0 = \sin n\pi = 0$, the diffracted H_z-wave satisfies the boundary condition on the ground plane. If we write the total H_z-field in the exterior region as

$$H_z^2 = H_z^{inc} + H_z^{ref} + H_z^{dif},$$

(69)

it is clear that $\partial_\phi H_z^2 = 0$ for $\phi = 0, \pi$; the total exterior H_z-field satisfies the boundary condition on the perfectly reflecting ground plane.

We now proceed to write the incident and reflected H_z-fields as series over $n = 0$ to ∞ rather than over all n. We write the incident field as

$$H_z^{inc} = -J_0(kr) + \sum_{n=0}^{\infty} i^n J_n(kr)\, e^{-in(\phi - \phi^{inc})} + \sum_{n=0}^{\infty} i^n J_n(kr)\, e^{in(\phi - \phi^{inc})},$$

(70)

or

$$H_z^{inc} = -J_0(kr) + 2\sum_{n=0}^{\infty} i^n J_n(kr)\, \cos n\left(\phi - \phi^{inc}\right).$$

(71)

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Similarly for the reflected field

$$H_z^{ref} = -J_0(kr) + 2\sum_{n=0}^{\infty} i^n J_n(kr) \cos n \left(\phi + \phi^{inc}\right).$$ (72)

Hence we can write, using the identiy $\cos n \left(\phi \pm \phi^{inc}\right) = \cos n\phi \cos n\phi^{inc} \mp \sin n\phi \sin n\phi^{inc}$,

$$H_z^{inc} + H_z^{ref} = -2J_0(kr) + 4\sum_{n=0}^{\infty} i^n J_n(kr) \cos n\phi \cos n\phi^{inc}.$$ (73)

Recalling our expression for the diffracted field, we write the total exterior H_z-field as

$$H_z^2 = -2J_0(kr) + \sum_{n=0}^{\infty} \left[4 i^n J_n(kr) \cos n\phi^{inc} + A_n H_n^{(2)}(kr) \right] \cos n\phi.$$ (74)

3.3   Interior Field

In the interior region the total H_z-field can be written as

$$H_z^1 = \sum_{n=-\infty}^{\infty} D_n J_n(kr) e^{in\phi},$$ (75)

where D_n is an unknown modal coefficient. However, for this calculation, it will be more convenient if we write this equivalently as

$$H_z^1 = \sum_{n=0}^{\infty} J_n(kr) \left(B_n \cos n\phi + C_n \sin n\phi\right),$$ (76)

where B_n , C_n are the modal coefficients that will be determined from the boundary conditions.

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3.4 Electric Field Component

The ϕ-component of the electric field is also of interest in this problem. From Maxwell's equations we have

$$E_\phi = \frac{i}{\omega\varepsilon}\,\partial_r H_z,$$
(77)

so that

$$E_\phi^2 = \frac{ik}{\omega\varepsilon}\left\{ -2\,J_0'(kr) + \sum_{n=0}^{\infty}\left[4\,i^n J_n'(kr)\cos n\phi^{inc} + A_n H_n^{(2)'}(kr)\right]\cos n\phi \right\}.$$
(78)

$$E_\phi^1 = \frac{ik}{\omega\varepsilon}\sum_{n=0}^{\infty} J_n'(kr)\,(B_n\cos n\phi + C_n\sin n\phi).$$
(79)

and, for clarity

$$E_\phi^{dif} = \frac{ik}{\omega\varepsilon}\sum_{n=0}^{\infty} A_n H_n^{(2)'}(kr)\cos n\phi.$$
(80)

3.5 Boundary Conditions

Boundary conditions required for solving this TE problem are

$$E_\phi^1 = 0 \text{ for } r = a \text{ and } \pi < \phi < 2\pi,$$
(81)

$$E_\phi^1 = E_\phi^2 \text{ for } r = a \text{ and } 0 < \phi < \pi,$$
(82)

$$H_z^1 = H_z^2 \text{ for } r = a \text{ and } 0 < \phi < \pi.$$
(83)

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The first condition enforces zero tangential electric field on the surface of the crack and the last two ensure continuous tangential electric and magnetic fields across the aperture. Recall that the total exterior field, as written, satisfies the boundary conditions along the ground plane.

Applying the boundary conditions, we get the following three equations

$$\sum_{n=0}^{\infty} B_n J'_n(ka) \cos n\phi_1 + \sum_{n=0}^{\infty} C_n J'_n(ka) \sin n\phi_1 = 0, \tag{84}$$

$$\sum_{n=0}^{\infty} B_n J'_n(ka) \cos n\phi_2 + \sum_{n=0}^{\infty} C_n J'_n(ka) \sin n\phi_2 \tag{85}$$

$$= -2 J'_0(ka) + \sum_{n=0}^{\infty} \left[4 i^n J'_n(ka) \cos n\phi^{inc} + A_n H_n^{(2)'}(ka) \right] \cos n\phi_2 .$$

$$\sum_{n=0}^{\infty} B_n J_n(ka) \cos n\phi_2 + \sum_{n=0}^{\infty} C_n J_n(ka) \sin n\phi_2 \tag{86}$$

$$= -2 J_0(ka) + \sum_{n=0}^{\infty} \left[4 i^n J_n(ka) \cos n\phi^{inc} + A_n H_n^{(2)}(ka) \right] \cos n\phi_2$$

where $\pi < \phi_1 < 2\pi$ and $0 < \phi_2 < \pi$. We now define the following for convenience

$$F_n = \left\{ \left(4 i^n \cos n\phi^{inc} - B_n \right) J_n(ka) + A_n H_n^{(2)}(ka) \right\}, \tag{87}$$

$$G_n = -B_n J_n(ka), \tag{88}$$

$$R_n = C_n J_n(ka) \tag{89}$$

as well as $\dfrac{\partial F_n}{\partial (ka)} = F'_n$ and so on. The three Boundary Condition equations thus become

$$\sum_{n=1}^{\infty} (-1)^n G'_n \cos n\phi = \sum_{n=1}^{\infty} (-1)^n R'_n \sin n\phi + B_0 J'_0(ka), \tag{90}$$

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where we have made the change of variables $\phi_1 \rightarrow \phi + \pi$ and $\phi_2 \rightarrow \phi$.

$$\sum_{n=1}^{\infty} F'_n \cos n\phi = \sum_{n=1}^{\infty} R'_n \sin n\phi + (2 - B_0) J'_0(ka) - A_0 H_0^{(2)'}(ka). \tag{91}$$

$$\sum_{n=1}^{\infty} F_n \cos n\phi = \sum_{n=1}^{\infty} R_n \sin n\phi + (2 - B_0) J_0(ka) - A_0 H_0^{(2)}(ka). \tag{92}$$

All three equations are valid for $0 < \phi < \pi$ and all are of the same form with respect to ϕ.

3.6   Orthogonality

We consider the equation

$$\sum_{n=1}^{\infty} F_n \cos n\phi = \sum_{n=1}^{\infty} R_n \sin n\phi + K(ka), \tag{93}$$

where F_n, R_n, $K(ka)$ are not functions of ϕ and $0 < \phi < \pi$. We have the following orthogonality relations among the sinusoids

$$\int_0^\pi \cos n\phi \cos m\phi \, d\phi = \frac{\pi}{2} \delta_{mn} \qquad (m, n \geq 1), \tag{94}$$

and

$$\int_0^\pi \sin n\phi \cos m\phi \, d\phi = \begin{cases} \dfrac{2n}{n^2 - m^2} & \text{if } n - m \text{ is odd} \\[2mm] 0 & \text{if } n - m \text{ is even} \end{cases} \tag{95}$$

We also note that

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$$\int_0^\pi \cos m\phi \, d\phi = 0 \qquad m = 1, 2, 3 \dots \quad . \tag{96}$$

Now, we multiply Eq. (93) by $\cos m\phi$ and integrate from 0 to π.

$$\int_0^\pi \sum_{n=1}^\infty F_n \cos n\phi \cos m\phi \, d\phi = \int_0^\pi \sum_{n=1}^\infty R_n \sin n\phi \cos m\phi \, d\phi + K \int_0^\pi \cos m\phi \, d\phi. \tag{97}$$

The above relations allow us to write

$$F_m = \frac{4}{\pi} \sum_n^\bullet \left(\frac{n R_n}{n^2 - m^2} \right) \quad (m \geq 1). \tag{98}$$

as a solution to Eq. (93). Here $\sum_n^\bullet$ indicates sum over n from 1 to ∞ for those values of n where $(n - m)$ is odd.

The case $m = 0$ is considered separately. In this case we effectively multiply Eq. (93) by unity and then integrate from 0 to π. We note

$$\int_0^\pi \sin n\phi \, d\phi = \begin{cases} \dfrac{2}{n} & n \text{ odd} \\[2mm] 0 & n \text{ even} \end{cases} . \tag{99}$$

We then have, in addition to Eq. (98) the relation

$$\sum_{n=1,3,5}^\infty \frac{1}{n} R_n = -\frac{\pi}{2} K \quad (m = 0). \tag{100}$$

The three Boundary Condition equations thus have the solutions, for $m \geq 1$,

$$G'_m = -\frac{4}{\pi} \overset{\bullet}{\underset{n}{\sum}} \left(\frac{n R'_n}{n^2 - m^2} \right).$$

(101)

$$F'_m = \frac{4}{\pi} \overset{\bullet}{\underset{n}{\sum}} \left(\frac{n R'_n}{n^2 - m^2} \right).$$

(102)

$$F_m = \frac{4}{\pi} \overset{\bullet}{\underset{n}{\sum}} \left(\frac{n R_n}{n^2 - m^2} \right).$$

(103)

or, recalling our previous definitions

$$B_m J'_m(ka) = \frac{4}{\pi} \overset{\bullet}{\underset{n}{\sum}} \left(\frac{n C_n J'_n(ka)}{n^2 - m^2} \right).$$

(104)

$$\left(4 i^m \cos m\phi^{inc} - B_m \right) J'_m(ka) + A_m H_m^{(2)'}(ka) = \frac{4}{\pi} \overset{\bullet}{\underset{n}{\sum}} \left[\frac{n C_n J'_n(ka)}{n^2 - m^2} \right].$$

(105)

$$\left(4 i^m \cos m\phi^{inc} - B_m \right) J_m(ka) + A_m H_m^{(2)}(ka) = \frac{4}{\pi} \overset{\bullet}{\underset{n}{\sum}} \left[\frac{n C_n J_n(ka)}{n^2 - m^2} \right].$$

(106)

For $m = 0$ we have

$$\overset{\infty}{\underset{n=1,3,5}{\sum}} \frac{1}{n} C_n J'_n(ka) = \frac{\pi}{2} B_0 J'_0(ka).$$

(107)

$$\overset{\infty}{\underset{n=1,3,5}{\sum}} \frac{1}{n} C_n J'_n(ka) = \frac{\pi}{2} \left\{ (2 - B_0) J'_0(ka) + A_0 H_0^{(2)'}(ka) \right\}.$$

(108)

$$\overset{\infty}{\underset{n=1,3,5}{\sum}} \frac{1}{n} C_n J_n(ka) = \frac{\pi}{2} \left\{ (2 - B_0) J_0(ka) + A_0 H_0^{(2)}(ka) \right\}.$$

(109)

These two sets of equations will be used to solve for the modal coefficients A_n, B_n, and C_n.

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3.7 Solution for Modal Coefficients

We first eliminate A_m and A_0 respectively from the two sets of equations above, and then we use the Wronskian relationship, $W\left[J_m(ka)H_m^{(2)}(ka)\right] = -\dfrac{2i}{\pi\,ka}$, to write

$$0 = \frac{4}{\pi}\sum_n^{\bullet}\left(\frac{nC_n}{n^2-m^2}\right)\left[J_n'(ka)H_m^{(2)}(ka)-J_n(ka)H_m^{(2)'}(ka)\right] \tag{110}$$

$$+\frac{2i}{\pi\,ka}B_m-\frac{8}{\pi}\left(\frac{i}{ka}\right)i^m\cos m\phi^{inc},$$

where we have now combined the two sets of equations. Substituting for B_m, and rearranging, we write, for $m = 0, 1, 2, 3...$

$$\sum_n^{\bullet}\left(\frac{nC_n}{n^2-m^2}\right)\left\{J_n'(ka)-\frac{\pi\,ka}{2i}\left[J_m'(ka)\;J_n'(ka)H_m^{(2)}(ka)-J_n(ka)\;J_m'(ka)H_m^{(2)'}(ka)\right]\right\} \tag{111}$$

$$= \pi\,i^m J_m'(ka)\cos m\phi^{inc}.$$

which is an expression that we can solve for C_n. Then, when we have C_n we can write

$$A_0 = \left[\frac{4}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}C_n J_n'(ka)-2\,J_0'(ka)\right]\frac{1}{H_0^{(2)'}(ka)}, \tag{112}$$

and, for $l = 1, 2, 3 ...$

$$A_l = \left[\frac{8}{\pi}\sum_n^{\bullet}\left(\frac{nC_n J_n'(ka)}{n^2-l^2}\right)-4\,i^l J_l'(ka)\cos l\phi^{inc}\right]\frac{1}{H_l^{(2)'}(ka)}. \tag{113}$$

3.8 Small Crack Limit

When $ka \to 0$ the cylinder functions have the following simple forms

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$$J_l(ka) \rightarrow \frac{(ka/2)^l}{l!},$$

(114)

and

$$H_l^{(2)}(ka) \rightarrow \frac{i}{l\pi}\frac{l!}{(ka/2)^l} \quad l \neq 0; \quad H_0^{(2)}(ka) \rightarrow \frac{2}{i\pi}\ln(ka).$$

(115)

The equations that we must solve also become much simpler, and we can separate the ka-dependence from the necessary numerical matrix inversion.

When the radius of the crack is small compared to the wavelength of the incident TE wave we have, setting $m = 0$ in Eq. (111) and then substituting into Eq. (112),

$$A_0 = i\frac{\pi}{2}(ka)^2\left[1 - \frac{2}{1 + \frac{1}{2}(ka)^2\ln(ka)}\right],$$

(116)

and, with $\varepsilon = ka/2 \ll 1$ and $c_n = C_n J'_n(ka)$,

$$A_l = 8\,i\varepsilon\frac{\varepsilon^l}{l!}\left[\sum_n\left(\frac{2nc_n}{n^2 - l^2}\right) - \frac{\pi}{2}i^l\frac{l}{l!}\frac{\varepsilon^l}{\varepsilon}\cos l\phi^{inc}\right] \quad (l \geq 1),$$

(117)

where the c_n are found from

$$\sum_n\left(\frac{3n + m}{n^2 - m^2}\right)c_n = \pi i^m\frac{m}{m!}\frac{\varepsilon^m}{\varepsilon}\cos m\phi^{inc} \quad (m, n \geq 1).$$

(118)

If we define

$$f_m = \pi i^m\frac{m}{m!}\cos m\phi^{inc},$$

(119)

$$T_{mn} = \frac{3n + m}{n^2 - m^2},$$

(120)

we have

$$\sum_n \dot{T}_{mn} c_n = f_m \varepsilon^{m-1} \quad (m, n \geq 1).$$ (121)

Hence, we write

$$A_l = 4 i \frac{\varepsilon^l}{l!} \left[\sum_n \sum_m \frac{4 n T_{nm}^{-1} \dot{f}_m \varepsilon^m}{n^2 - l^2} - \dot{f_l} \varepsilon^l \right] = 4 i \frac{\varepsilon^l}{l!} \sum_m \mathcal{Q}_{lm} \dot{f}_m \varepsilon^m.$$ (122)

where

$$\mathcal{Q}_{lm} = \sum_n^{\bullet} \frac{4 n T_{nm}^{-1}}{n^2 - l^2} - \delta_{lm}.$$

For small ε (that is, cracks much narrower than a wavelength) this shows that the only significant A_l are A_0, A_1, with A_1 given approximately by

$$A_1 \approx -4 \pi \varepsilon^2 \mathcal{Q}_{11} \cos \phi^{inc} = -\mathcal{Q}_{11} \pi (ka)^2 \cos \phi^{inc}.$$ (123)

We see from the above that the diffracted field behaves as $(ka)^2$ for small ka,

$$H_z^{dif} \approx -\frac{\pi}{2} (ka)^2 \left[i \left(-1 + \frac{2}{1 + \frac{1}{2}(ka)^2 \ln (ka)} \right) H_0^{(2)}(kr) + 2 \mathcal{Q}_{11} H_1^{(2)}(kr) \cos \phi \cos \phi^{inc} \right].$$ (124)

If we use the far-field expressions for the Hankel functions, we write for plane TE wave scattering from a small cylindrical crack $(r \to \infty)$

$$H_z^{dif} \approx -\sqrt{\frac{\pi}{2}} (ka)^2 e^{i3\pi/4} \frac{e^{-ikr}}{\sqrt{kr}} \left[-1 + \frac{2}{1 + \frac{1}{2}(ka)^2 \ln (ka)} + 2 \mathcal{Q}_{11} \cos \phi \cos \phi^{inc} \right].$$ (125)

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where

$$E_\phi^{dif} \sim \sqrt{\frac{\varepsilon}{\mu}} \ H_z^{dif}. \tag{126}$$

Performing the necessary matrix inversion we find that $Q_{11} \approx 0.18$ which gives for TE far-field scattering from a small crack

$$H_z^{dif} \approx \sqrt{2\pi} \ (ka)^2 \ e^{i3\pi/4} \ \frac{e^{-ikr}}{\sqrt{kr}} \left[\frac{1}{2} - \frac{1}{1 + \frac{1}{2} (ka)^2 \ln (ka)} - 0.18 \cos \phi \cos \phi^{inc} \right]. \tag{127}$$

A computer program has been written to check the consistency of the small crack approximation. The general expressions, [Eqs. (111) - (113)], are solved numerically and then compared with the *small-crack* results. Figure 4 shows the negative of the bracketed quantities from Eq. (116) compared to the general-ka expressions as ka becomes small. Note that for ka very small, the bracketed terms in Eq. (116) tend to negative unity. Figure 5 shows the coefficient Q_{11} compared to the lowest mode coefficient from the general program as ka becomes small. For small values of ka, [where $(ka)^2$ can be neglected compared to unity] the lowest mode dominates, and the general and small-crack coefficients of this mode agree.

The problem of plane wave scattering from a slit in an infinitely thin conducting plane has been solved and is well known. For the TM polarization, we found the results for scattering from the small semicircular crack to be in agreement with those for the slit (differing by a constant factor.) For TE scattering from a narrow slit, Reference [14] gives

$$H_z^{dif} = \sqrt{2\pi} \ e^{i3\pi/4} \ \frac{e^{-ikr}}{\sqrt{kr}} \left[\frac{1 + .25(ka)^2 \left(\sin^2 \phi^{inc} - \cos^2 \phi \right)}{2 \ln (ka/4) + 2\gamma + i\pi} + 0.25 (ka)^2 \cos \phi \cos \phi^{inc} \right], \tag{128}$$

where γ is Euler's constant. We note that for the TE polarization, the leading term for the small crack is $(ka)^2$ while the leading term for the slit is $\frac{1}{\ln (ka)}$ and we conclude that for the TE polarization the narrow slit scatters more than the small crack. The physical reasoning is as follows. For TM polarization, the currents excited by the incident electric field are normal to the crack/slit. In the former case the small crack and the small slit appear to the excited currents to be similar, while in the latter case the currents are stopped completely by the *open circuit* of the slit, but not by the perfectly conducting crack.
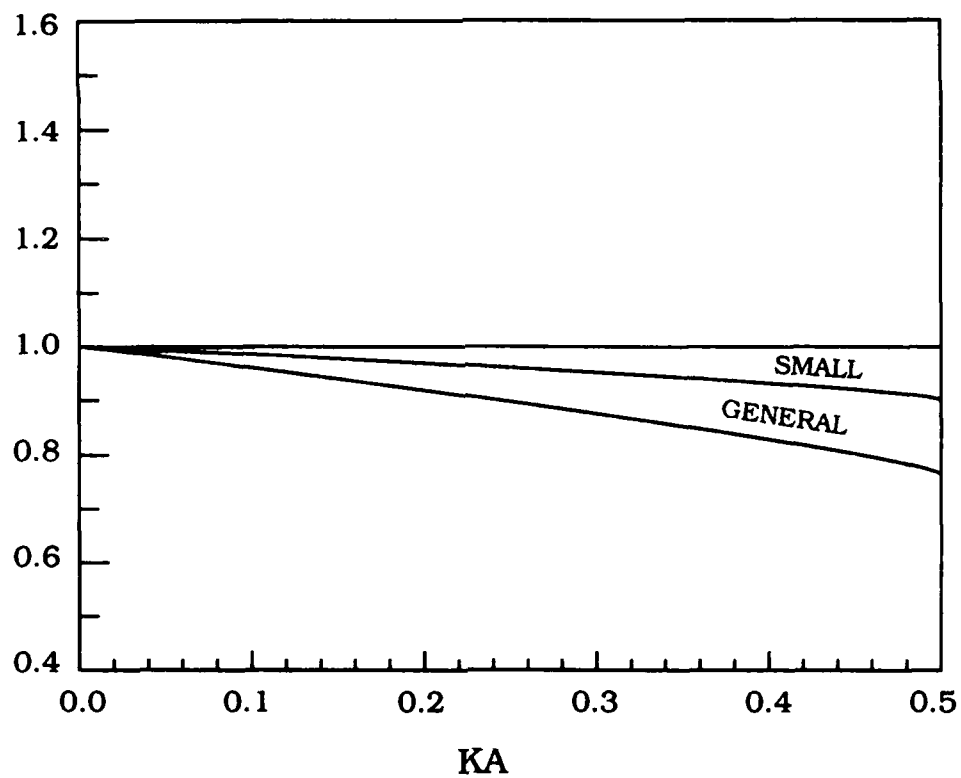
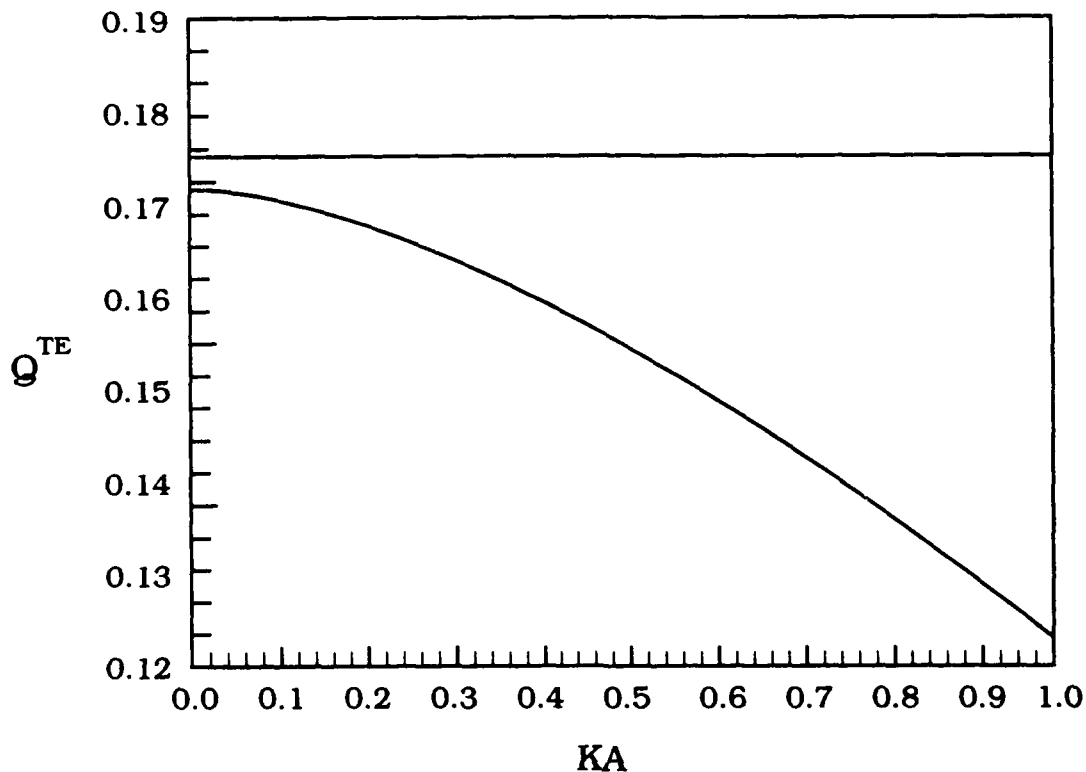Figure 4. General and Small-Crack Coefficients of A_0^{TE}.

0.19

0.18

0.17

Q^{TE} 0.16

0.15

0.14

0.13

0.12

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

KA

Figure 5. General and Small-Crack Q^{TE} vs. ka.

4. SUMMARY

In this report we have developed expressions for the fields scattered from a semicylindrical crack in a perfectly conducting ground plane for both transverse magnetic and transverse electric polarizations. Although a numerical matrix inversion is required in the solution, when the crack is much narrower than the wavelength of the incident radiation, the ka-dependence is removed from the numerical matrix inversion. The closed-form results that are found for small cracks are thus functions of ka and are valid for any value of ka such that ka^2 can be neglected compared to unity.

Since the TM and TE results for small cracks are functionally dependent upon ka, they can be combined vectorially to consider off-normal scattering from small cracks. Because of this, the results presented here for scattering from a small straight crack can be extended by such methods as Incremental Diffraction Coefficients (IDC) [14] to calculate the scattering from a small curved crack. Large reflector antennas are typically assembled from several sections, leaving gaps between them. Aircraft have gaps around doors and access panels. The former

affect the radiation pattern of the antenna and the latter contribute to the aircraft's radar cross section. Using IDC, Shore and Yaghjian [14] have modeled these as slits in thin conducting screens, but the geometry of a cylindrical crack seems a more physically realistic model. Since the *crack* and *slit* only agree well for the (small ka) TM polarization, real cracks may be better approximated using the results presented here.

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RADC plans and executes research, development, test and selected acquisition programs in support of Command, Control, Communications and Intelligence (C^3I) activities. Technical and engineering support within areas of competence is provided to ESD Program Offices (POs) and other ESD elements to perform effective acquisition of C^3I systems. The areas of technical competence include communications, command and control, battle management information processing, surveillance sensors, intelligence data collection and handling, solid state sciences, electromagnetics, and propagation, and electronic reliability/maintainability and compatibility.